



# Elimination of harmonics in multilevel inverters with non-equal dc sources using PSO

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## ABSTRACT

Multilevel inverters supplied from equal and constant dc sources almost do not exist in practical applications. The variation of the dc sources affects the values of the switching angles required for each specific harmonic profile, as well as increases the difficulty of the harmonic elimination's equations. This paper presents an extremely fast optimal solution of harmonic elimination of multilevel inverters with non-equal dc sources using a novel Particle Swarm Optimization (PSO) algorithm. The overall system is suitable for large variable speed drives, UPS systems, and on-line utility applications such as static var compensation. A set of mathematical equations describing the general output waveform of the multilevel inverter with non-equal dc sources is formulated. PSO is then employed to compute the optimal solution set of switching angles, if it exists, for each required harmonic profile. Theoretical studies for different case studies regarding the number of levels and harmonic profile are carried out to show the effectiveness and robustness of the proposed technique, and validated through both simulations and laboratory experimentation.

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## 1. Introduction

Multilevel inverter is considered as one of the most significant recent advances in power electronics [1]. Multilevel inverters have drawn tremendous interest in the field of high-voltage high-power applications such as laminators, mills, conveyors, compressors, large induction motor drives, UPS systems, and static var compensation. Its concept is based on producing small output voltage steps, resulting in better power quality. Despite the need for more power transistors, they operate at low voltage levels and also at low switching frequency so that the switching losses are also reduced. Other advantages include better electromagnetic compatibility due to the low  $dv/dt$  transitions.

Some of the fundamental multilevel topologies include the diode-clamped [2], flying capacitor [3], and cascaded H-bridge structures [4]. Recently, with the increase in the rating of the available switches, there has been much interest in new topologies aiming to reduce the amount of semiconductor devices [5]. Multilevel inverters are mostly supplied from dc sources obtained from fuel cells, ultra capacitors, etc. It is worth noting that in most of the reported work, it was assumed that the dc sources were all equal, which will probably not be the case in applications even if the sources are nominally equal.

The key issue in designing an effective multilevel inverter is to ensure that the total harmonic distortion (THD) of the output voltage waveform is within acceptable limits. Selective harmonic elimination pulse width modulation (SHE-PWM) has been intensively studied in order to achieve low THD [6]. The output voltage waveform analysis using Fourier theory produces a set of non-linear transcendental equations. The solution of these equations, if exists, gives the switching angles required for certain fundamental component and selected harmonic profile. Iterative procedures such as Newton–Raphson method has been used to solve these sets of equations [7]. This method is derivative-dependent and may end in local optima, and a judicious choice of the initial values alone guarantees conversion [8]. Another approach based on converting the transcendental equation into polynomial equations is presented in [9,10], where resultant theory is applied to determine the switching angles to eliminate specific harmonics. That approach, however, appears to be unattractive because as the number of inverter levels increases, so does the degree of the polynomials of the mathematical model. This is likely to lead to numerical difficulty and substantial computational burden as well.

Genetic algorithms (GA) have been used to obtain optimal solutions for inverter circuits supplied from constant dc sources [11,12]. Despite their effectiveness in selective harmonic elimination, they are complicated and their parameters such as crossover and mutation probability, population size and number of generations are usually selected as common values given in literature or by means of a trial and error process to achieve the best solution set.

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Heuristic algorithms such as Particle Swarm Optimization (PSO) [13] have the ability to combat the above drawbacks. As an optimization technique, PSO is much less dependent on the start values of the variables in the optimization problem when compared with the widely used Newton–Raphson or mathematical programming techniques such as Sequential Quadratic Programming (SQP). In addition, PSO does not rely on the guidance of the gradient information, such as the Jacobian matrix, hence it is more capable of determining the global optimum solution. PSO deal with all problems that usually considered very hard for researchers, such as integer variables, non-convex functions, non-differentiable functions, domains not connected, badly behaved functions, multiple local optima, and multiple objectives [14,15]. For these reasons, PSO has been adopted in this study.

This paper presents an optimal minimization technique assisted with PSO algorithm in order to highly reduce the computational burden associated with the solution of the non-linear transcendental equations of the harmonic elimination problem of a cascaded H-bridge inverter with non-equal dc sources. The presented method has the advantage of its extremely short time required to reach at the optimal solution, if existed, as this is essential for on-line updates such that the algorithm can cope with any sudden variations of the voltage levels of the dc sources. The presented method can be extended to other multilevel inverter topologies, where an accurate and fast solution is guaranteed even for large number of levels and switching angles. Problem formulation and analysis are presented, simulations of the overall system for different case studies are carried out, and experimental verifications are also conducted and compared to those from both simulations and conventional methods, where the superiority of the presented algorithm is reported.

**2. Cascaded H-bridge multilevel inverter**

The cascaded H-bridge multilevel inverter consists of a series of single-phase full-bridge (H-bridge) inverter units, as shown in Fig. 1. It is modular in nature and can be extended to any required

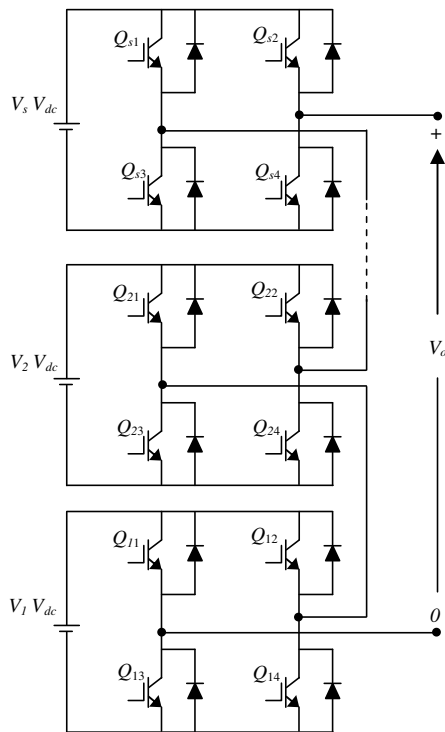


Fig. 1. Single-phase structure of a multilevel cascaded H-bridge inverter.

number of levels. It is supplied from several separate dc sources (SDCSs), which may be obtained from batteries, fuel cells, solar cells, or ultra-capacitors [16]. Each SDCS is connected to a single-phase full-bridge inverter and can generate three different voltage outputs, + $V_{dc}$ , 0, and  $-V_{dc}$ . This is accomplished by connecting the dc source to the ac output side by using different combinations of the four switches  $Q_1, Q_2, Q_3,$  and  $Q_4$ . The ac outputs of the modular full-bridge inverters are connected in series such that the synthesized voltage waveform is the sum of all of the individual inverter outputs. All semiconductor devices of the H-bridges are only switching at the fundamental frequency, and consequently this is referred to as the fundamental switching scheme. Also, each H-bridge unit generates a quasi-square waveform by phase-shifting its positive and negative phase legs' switching timings. The number of output voltage levels in a cascaded multilevel inverter is then  $2s + 1$ , where  $s$  is the number of dc sources. Three-phase version of this circuit is also available by adding another two phases and connecting their neutral point together. Fig. 2 shows the generalized output voltage of cascaded H-bridge multilevel inverter with non-equal dc sources. The total output voltage is given by  $v_o = v_1 + v_2 + v_i + \dots + v_s$ . With enough levels and an appropriate switching angles  $\theta_1, \theta_2, \theta_i, \dots, \theta_s$ , the multilevel inverter results in an output voltage that is almost sinusoidal with a low THD (<5%) with each of the active devices subjected to a single dc source, and only switching at the fundamental frequency. This reduces both the voltage stress and the switching losses of the semiconductor devices, resulting in a better utilization and high overall efficiency.

**3. Mathematical method of switching**

Since the generalized output waveform of multilevel inverters shown in Fig. 2 is non-sinusoidal, therefore it may be expressed in Fourier series expansion in its general form as:

$$v_{out}(\theta) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\theta + B_n \sin n\theta) \tag{1}$$

where

$$A_0 = \frac{1}{2\pi} \int_0^{2\pi} f(t) dt, \quad A_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin n\omega t dt, \\ B_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos n\omega t dt \tag{2}$$

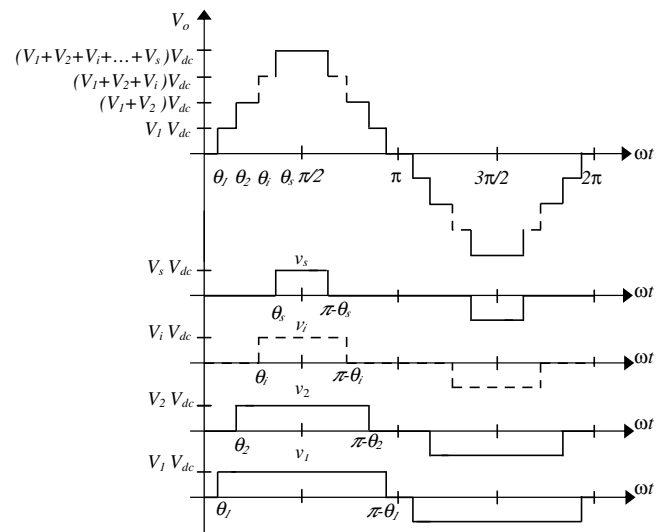


Fig. 2. Generalized output waveform of a cascaded multilevel inverter.

Since all inverter output voltages have no dc component or average value, therefore  $A_0 = 0$ . Also, as the general waveform shown in Fig. 2 possesses an odd function or point symmetry, i.e.  $f(t) = -f(-t)$ , and mirror or half- and quarter-wave symmetry i.e.  $f(t) = -f(t + 90^\circ)$ , only sinusoidal terms with odd order harmonics exit in the expansion. Then Eq. (1) is reduced to:

$$v_{out}(\theta) = \sum_{n=1,3,5,\dots}^{\infty} B_n \sin n\theta \quad (3)$$

Assuming that the non-equal dc sources are known, and taking into consideration the abovementioned characteristics of the inverter waveform shown in Fig. 2, the Fourier series expansion of the generalized stepped output voltage waveform of the multilevel inverter with non-equal dc sources can be expressed as:

$$V_o(\omega t) = \sum_{n=1,3,5,\dots}^s \frac{4V_{dc}}{n\pi} (V_1 \cos(n\theta_1) + V_2 \cos(n\theta_2) + \dots + V_s \cos(n\theta_s)) \sin(\omega t) \quad (4)$$

where  $s$  is the number of dc sources, and the product  $V_i V_{dc}$  is the value of the  $i$ th dc source. If all the dc sources have the same value of  $V_{dc}$ , then  $V_1 = V_2 = \dots = V_s = 1$ . Eq. (4) has  $s$  variables  $(\theta_1, \theta_2, \theta_3, \dots, \theta_s)$ , where  $0 \leq \theta_1 < \theta_2 < \dots < \theta_s \leq \pi/2$ , and a solution set is obtained by assigning a specific value to the fundamental component,  $V_f$ , and equating  $s - 1$  harmonics to zero as given below:

$$\left. \begin{aligned} V_1 \cos(\theta_1) + V_2 \cos(\theta_2) + \dots + V_s \cos(\theta_s) &= m \\ V_1 \cos(3\theta_1) + V_2 \cos(3\theta_2) + \dots + V_s \cos(3\theta_s) &= 0 \\ V_1 \cos(5\theta_1) + V_2 \cos(5\theta_2) + \dots + V_s \cos(5\theta_s) &= 0 \\ \vdots & \\ V_1 \cos(n\theta_1) + V_2 \cos(n\theta_2) + \dots + V_s \cos(n\theta_s) &= 0 \end{aligned} \right\} \quad (5)$$

where  $m = V_f/(4V_{dc}/\pi)$ , and it is related to the modulation index  $m_i$  by  $m_i = m/s$ , where  $0 \leq m_i \leq 1$ .

An objective function is then needed for the optimization procedure, which is selected as a measure of effectiveness of eliminating selected order of harmonics while maintaining the fundamental component at a pre-specified value. Therefore, this objective function is defined as:

$$F(\theta_1, \theta_2, \dots, \theta_s) = \left[ \sum_{n=1}^s V_1 \cos(\theta_n) - m \right]^2 + \left[ \sum_{n=1}^s V_2 \cos(3\theta_n) \right]^2 + \dots + \left[ \sum_{n=1}^s V_s \cos((2s - 1)\theta_s) \right]^2 \quad (6)$$

The optimal switching angles are obtained by minimizing Eq. (6) subject to the constraint  $0 \leq \theta_1 < \theta_2 < \dots < \theta_s \leq \pi/2$ , and consequently the required harmonic profile is achieved. The main challenge is the non-linearity of the transcendental set of Eq. (5), as most iterative techniques suffer from convergence problems and other techniques such as elimination using resultant and GA are complicated.

#### 4. Solution using PSO

PSO has recently received much attention as robust stochastic search algorithms for optimization problem. PSO is an evolutionary computation technique developed by Kennedy and Eberhart in 1995 [17]. PSO combines social psychology principles in socio-cognition human agents and evolutionary operations. Based and inspired by social behavior of bird flocking or fish schooling, PSO conduct the searching process using a population of particles. A particle represents a potential solution to the problem under investigation. Each particle in a given population adjusts its position by

flying in a multi-dimensional search space until an unchanging position of the fittest particle is encountered.

Generally, PSO has the advantage of being very simple in concept, easy to implement, and computationally efficient algorithm. Unlike other heuristic algorithms, PSO possesses flexible and well-balanced operators to enhance and adapt the global and fine tune local search. PSO has been applied to various power system problems in which it proved to be extremely efficient relative to other evolutionary computation technique [18–22].

Like GA, PSO is a population based optimization tool. The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, are flown through the problem space by following the current optimum particles. Each particle keeps tracking its coordinates in the problem space which is associated with the best solution (fitness) it has achieved so far. The fitness of each particle is also stored or memorized. This best fitness is called *personal best*. Another best value that is tracked by the particle swarm optimizer is the best value obtained so far by any particle in the neighbors of the particle. When a particle takes all the population as its topological neighbors, the best value is a best of personal bests and is called *global best*.

Generally, the advantages of PSO may be summarized as in the following:

1. PSO is fairly easy to apply, and only few parameters are required to be adjusted before application.
2. PSO has no evolution intermediate operators (i.e. no crossover and mutation like the GA).
3. In PSO, the Global best particle (*Gbest*) leads the population by giving out information to the others potential solutions. Unlike GA, the whole population moves like one group [23].
4. PSO frequently converges to the solutions in fewer objective function evaluations than those required by GAs [23]. Therefore, PSO appears to be more efficient than GA.
5. PSO uses payoff (performance index) information along with memory to help and assist the search in the problem space.
6. In order to avoid premature convergence, PSO utilizes a distinctive feature of controlling a balance between global and local exploration of the search space. Such capability does not exist in GA.

PSO can be modeled mathematically by velocity and position equations as flows:

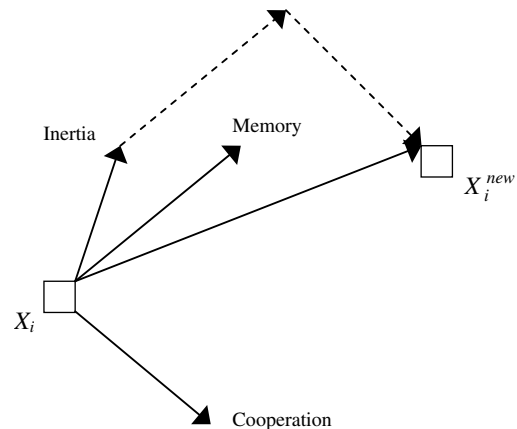


Fig. 3. Illustration of a particle movement influenced by three terms.

$$v_{ij}(k+1) = \phi(k)v_{ij}(k) + \alpha_1[\gamma_{1j}(p_{ij} - x_{ij}(k)) + \alpha_2\gamma_{2j}(G_{ij} - x_{ij}(k))] \quad (7)$$

$$x_{ij}(k+1) = x_{ij}(k) + v_{ij}(k+1) \quad (8)$$

where  $i$  is the particle index,  $j$  is the index of parameter of concern to be optimized,  $x$  is the position of the  $i$ th particle and  $j$ th param-

eter,  $k$  is the discrete time index,  $v$  is the velocity of the  $i$ th particle and  $j$ th parameter,  $P$  is the best position found by the  $i$ th particle and  $j$ th parameter (*personal best*),  $G$  is the best position found by swarm (*global best*),  $\gamma$  is a random uniform number between  $[0,1]$  applied to the  $i$ th particle,  $\phi$  is the inertia function,  $\alpha$  is the acceleration constants.

The first term represents the inertia or habit, where each particle continues moving in the direction it had previously moved. As training progresses, the influence of the past velocity becomes smaller. In this study, a decreasing linear inertia function  $\phi(k)$  is used. For the early stages of the search, a relatively large inertia is used to enhance the global exploration. On the other hand, when reaching the last stages, the inertia is reduced for better local exploitation. The second term represents the memory where each particle is attracted to the best point in its trajectory  $P$ . The third term represents a cooperation or information exchange, in which each particle is attracted to the best point found by all particles  $G$ . Fig. 3 illustrates the concept of particle movement influenced by the three terms.

PSO starts with a random initialization of  $N$  particles (positions). As an analogy with GA,  $N$  would best represent the population size [24,25]. Each particle is a row vector of a dimension  $l$  of randomly generated real values, where  $l$  is the number of parameters to be optimized. This is similar to chromosome length of a real-coded GA. In this study, the stopping criterion is either reaching the maximum number of iterations (1000 epoch), or reaching the

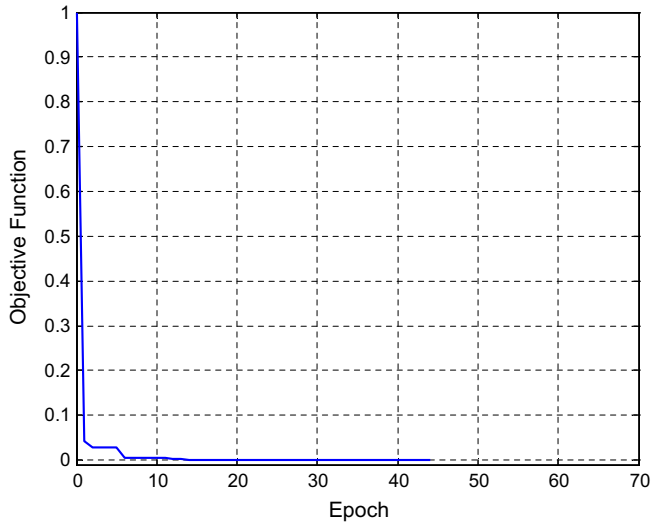


Fig. 4. Convergence characteristic of case 1.

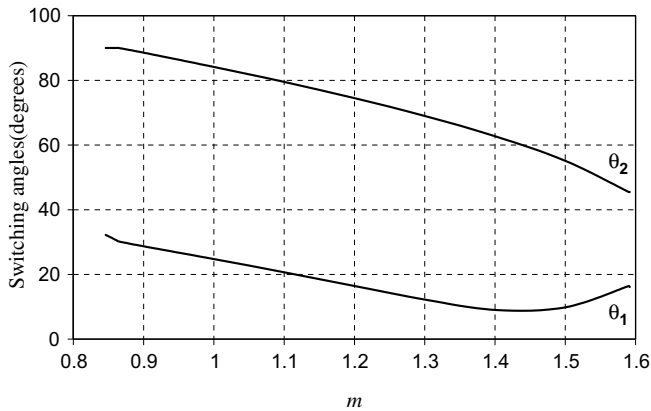


Fig. 5. Solutions for 2 angles versus  $m$  for 5-level inverter ( $V_1 = 1$  p.u.,  $V_2 = 0.9$  p.u., 3rd harmonic eliminated).

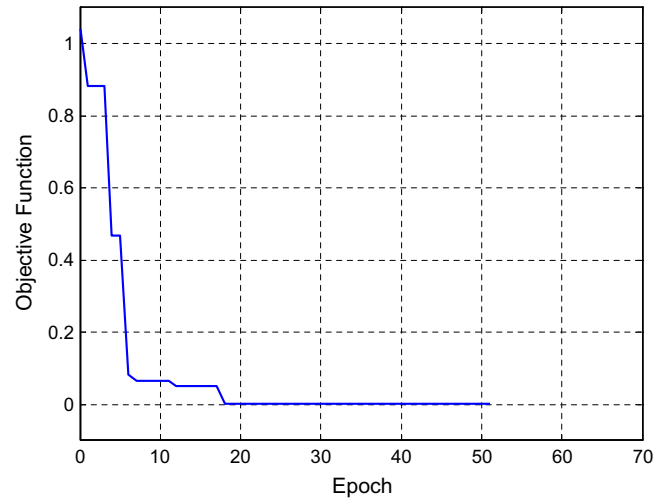


Fig. 7. Convergence characteristic of case 2.

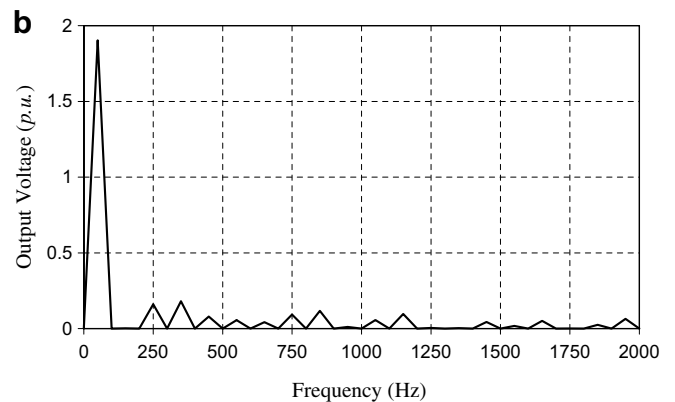
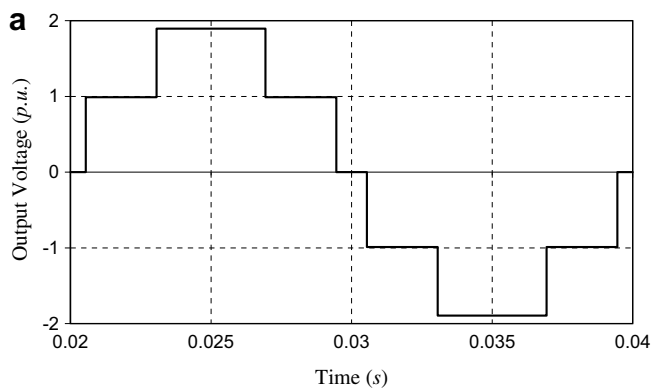


Fig. 6. (a) Output voltage, (b) corresponding FFT of 5-level inverter at  $V_1 = 1$  p.u.,  $V_2 = 0.9$  p.u., and  $m = 1.5$  (3rd harmonic eliminated).

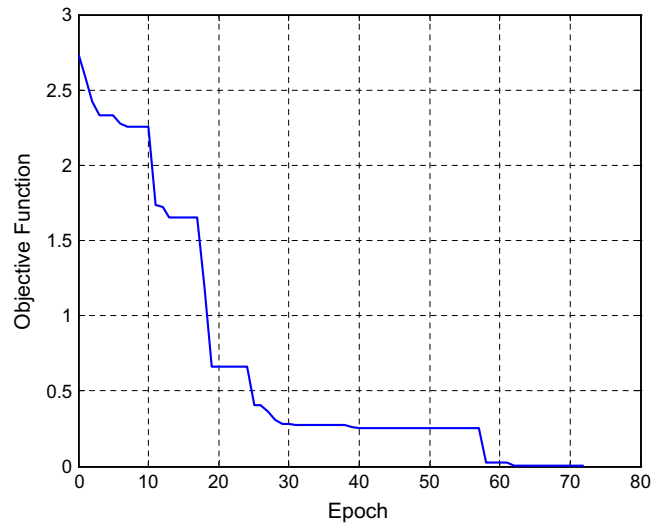
minimum global error gradient which has to satisfy  $|G(k) - G(k + 1)| < 10^{-9}$  to exit the training loop. It has been suggested that the acceleration constants  $\alpha_1$  and  $\alpha_2$  are both set to a fixed value of 2 [26]. The inertia function is a decreasing liner function with initial inertia weight of 0.9 and final weight of 0.2. The population size was chosen to be 40 particles.

**5. Simulation results**

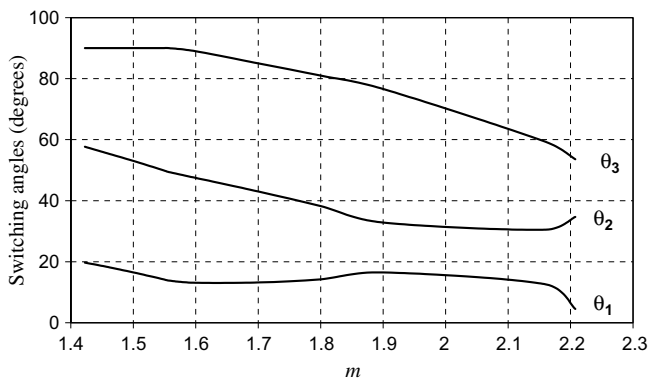
The generalized transcendental equations of multilevel inverter (5) are solved using the described PSO algorithm. The proposed technique has been applied to different study cases in order to confirm its ruggedness. The simulation results are obtained accordingly using Matlab [27]. It is assumed that the level of the non-equal dc sources can be measured, and  $V_{dc}$  has a nominal value of 1 p.u. and so does  $V_1$ , while the following sources will acquire different given values less than 1 p.u. The cases under consideration concern a single-phase system. However, this does not reduce the way the algorithm can be applied to the three-phase system. The location of harmonics to be eliminated varies between single- and three-phase cases since the triplen harmonics can be eliminated by the inverter structure, and there is no need to be included in the elimination process. For each inverter topology with a specific number of levels, a large number of solution sets can be obtained according to the values of  $m$  and the dc sources  $V_1, V_2, V_3, \dots$ . Therefore, different study cases will be presented for 5-, 7- and 9-level inverters to ensure the feasibility of the presented algorithm.

**5.1. Case 1: 5-level inverter;  $V_1 = 1$  p.u.,  $V_2 = 0.9$  p.u.**

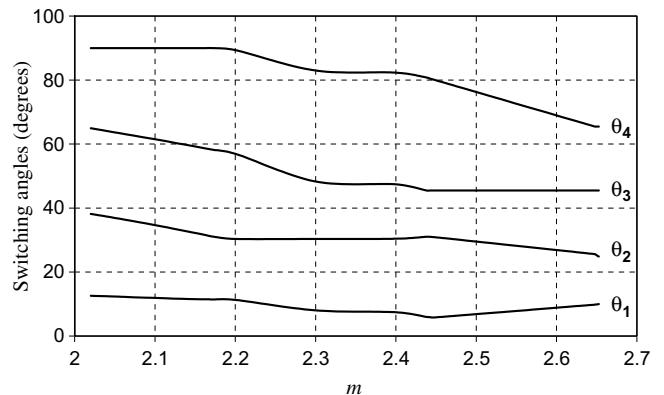
The proposed technique is applied to minimize the defined cost function for the above stated case. The convergence characteristic of the proposed PSO algorithm is depicted in Fig. 4. It is obvious that a near optimal solution is achieved by the PSO algorithm in



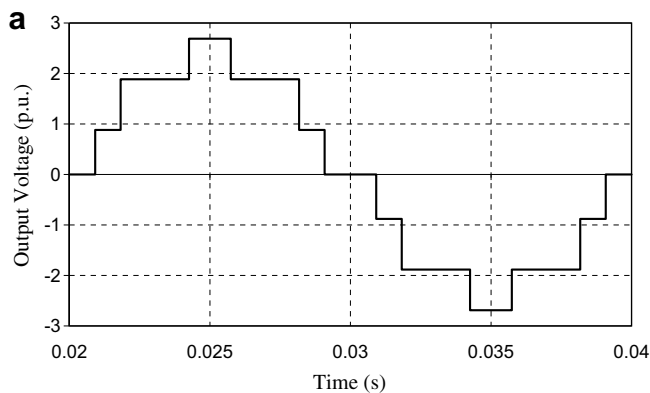
**Fig. 10.** Convergence characteristic of case 3.



**Fig. 8.** Solutions for 3 angles versus  $m$  for 7-level inverter ( $V_1 = 1$  p.u.,  $V_2 = 0.9$  p.u.,  $V_3 = 0.8$  p.u., 3rd and 5th harmonics eliminated).



**Fig. 11.** Solutions for 4 angles versus  $m$  for 9-level inverter (3rd, 5th, and 7th harmonics eliminated).



**Fig. 9.** (a) Output voltage, (b) corresponding FFT of 7-level inverter at  $V_1 = 1$  p.u.,  $V_2 = 0.9$  p.u.,  $V_3 = 0.8$  p.u., and  $m = 1.9$  (3rd and 5th harmonics eliminated).

about 11 iterations. The CPU execution time required for convergence is 1.27 s.

The PSO algorithm is used to find the switching angles for the abovementioned case. However, it is worth noting that the solution exists for a limited range of  $m$ , where  $0.84 \leq m \leq 1.59$ . Despite this is a natural phenomenon of multilevel inverters even with equal dc sources, the obtained range of  $m$  is wider than that obtained from conventional techniques. Fig. 5 illustrates the variation of the switching angles  $\theta_1$  and  $\theta_2$  versus  $m$ . As an example, an operating point when  $m = 1.5$  was chosen which sets the fundamental output voltage,  $V_f$ , to be 1.9 p.u. ( $s = 2$ ,  $m = 1.5$ ,  $V_{dc} = 1$  p.u.,  $V_f = (4mV_{dc}/\pi) = 1.9$  p.u.). For this point, the optimum values of the switching angles are:  $\theta_1 = 9.815^\circ$  and  $\theta_2 = 55.122^\circ$ . Fig. 6 shows the inverter output voltage and the corresponding harmonic spectrum at the abovementioned operating point. It is clear that the targeted 3rd harmonic is eliminated and the fundamental component is equal to 1.9 p.u. as desired.

5.2. Case 2: 7-level inverter;  $V_1 = 1$  p.u.,  $V_2 = 0.9$  p.u.,  $V_3 = 0.8$  p.u.

The proposed technique was applied to minimize the defined cost function for the above stated case. The convergence characteristic of the proposed method is depicted in Fig. 7. It is obvious from the convergence that a near optimal solution was achieved by PSO in about 18 iterations and approximately another 30 iterations was required to refine the solution. The total CPU execution time of PSO is 1.55 s.

Fig. 8 illustrates the variation of the switching angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  versus  $m$ , where it can be seen that solution exists in the range  $1.82 \leq m \leq 2.22$ . One particular operating point was chosen to demonstrate the effectiveness of the proposed method,  $m = 1.9$ , which sets the fundamental component  $V_f$  to 2.4 p.u. ( $s = 3$ ,  $m = 1.9$ ,  $V_{dc} = 1$  p.u.,  $V_f = (4mV_{dc}/\pi) = 2.419$  p.u.). For this point, the optimum values of the switching angles are:  $\theta_1 = 16.5014^\circ$ ,  $\theta_2 = 32.849^\circ$ , and  $\theta_3 = 76.6228^\circ$ . The inverter output voltage and its corresponding harmonic spectrum at the abovementioned operating point are depicted in Fig. 9, where it is clear that the 3rd and 5th harmonics are totally eliminated and the desired fundamental of 2.4 p.u. is achieved.

5.3. Case 3: 9-level inverter;  $V_1 = 1$  p.u.,  $V_2 = 0.9$  p.u.,  $V_3 = 0.8$  p.u.,  $V_4 = 0.7$  p.u.

In this case, four dc sources are considered to verify the feasibility and ruggedness of the proposed technique. The available four degrees of freedom offer the elimination of three low order harmonics and maintaining the fundamental at specific value. The proposed PSO technique was applied to minimize the cost function

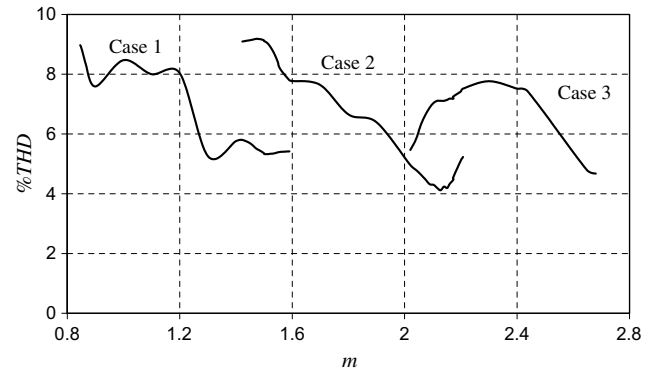


Fig. 13. The %THD versus  $m$  for different study cases.

Table 1

Comparison between iterative Newton–Raphson technique and the proposed PSO technique.

Case study	Iterative Newton–Raphson technique		Proposed PSO technique	
	Computational time (s)	%THD	Computational time (s)	%THD
1	18.02	8.01	1.27	5.44
2	20.45	9.52	1.55	6.40
3	24.26	10.26	3.14	7.20

for the above stated case. The convergence characteristic is shown in Fig. 10, where the optimal solution is reached after 62 iterations in 3.14 s.

Fig. 11 illustrates the variation of switching angles  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ , and  $\theta_4$  over the defined range of  $m$ ,  $2.02 \leq m \leq 2.66$ . An operating point of  $m = 2.4$  was chosen and applied to the inverter to indicate the effectiveness of the proposed technique to eliminate the 3rd, 5th, and 7th harmonics while maintaining the fundamental component  $V_f$  at 3.05 p.u. ( $s = 4$ ,  $m = 2.4$ ,  $V_{dc} = 1$  p.u.,  $V_f = (4mV_{dc}/\pi) = 3.05$  p.u.). For this point, the optimum values of the switching angles are:  $\theta_1 = 7.457^\circ$ ,  $\theta_2 = 30.4133^\circ$ , and  $\theta_3 = 47.4292^\circ$ , and  $\theta_4 = 82.3501^\circ$ . Fig. 12 shows the inverter output voltage and its corresponding harmonic spectrum at the abovementioned operating point, where the elimination of targeted harmonics is clearly evident and the fundamental is maintained at 3.05 p.u.

5.4. Performance index

In order to indicate the usefulness and effectiveness of the presented method, a quality factor is chosen as a performance index. The THD is very useful parameter to evaluate the performance of

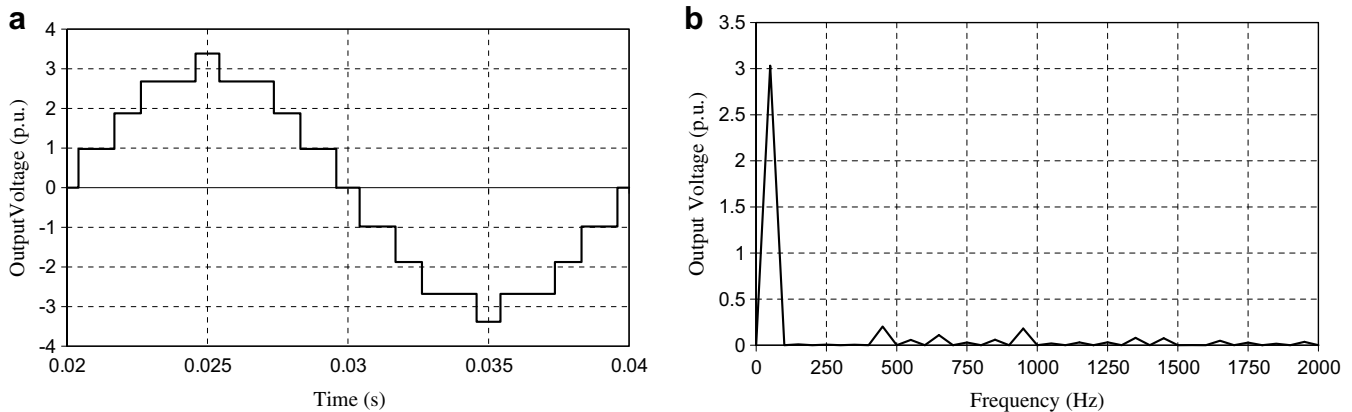


Fig. 12. (a) Output voltage, (b) corresponding FFT of 9-level inverter at  $V_1 = 1$  p.u.,  $V_2 = 0.9$  p.u.,  $V_3 = 0.8$  p.u.,  $V_4 = 0.7$  p.u., and  $m = 2.4$  (3rd, 5th, and 7th harmonics eliminated).



the inverter, and therefore it is considered in this work. The THD is defined as the total amount of harmonics relative to the fundamental, and can be calculated using Eq. (9) up to the 31st harmonic, where the inverter low pass filter typically eliminates other higher order harmonics.

$$\%THD = \frac{\sqrt{\sum_{n=3,5,\dots}^{31} V_n^2}}{V_1} \times 100 \tag{9}$$

Fig. 13 depicts the variation of the %THD versus  $m$  for the study cases under consideration. The obtained values of the %THD are better or at least the same as those obtained from other techniques [10]. Moreover, the computational time is also chosen as a secondary performance index to indicate the superiority of the proposed technique over conventional iterative techniques. The conventional Newton–Raphson iterative technique was used and applied to the same study cases mentioned earlier. The degree of accuracy was kept at 0.00001 while it was  $10^{-9}$  in the proposed PSO algorithm, and a limit of 1000 iterations was also set for halting as no optimal solution situation is highly expected as the inequality of the dc sources increases the complexity of the transcendental equations, and convergence problems are highly arise. The com-

parison between the conventional Newton–Raphson iterative method and the proposed PSO technique for the given study cases, regarding the computational time and corresponding %THD are tabulated in Table 1. It is clearly evident that the proposed PSO technique is both extremely faster and optimal than the conventional Newton–Raphson method, which reflects the superiority of the introduced technique as far as computational burdens and quality of the output waveform are concerned.

### 6. Experimental verification

A prototype single-phase cascaded H-bridge inverter was built using IRF630 (200 V, 9A) MOSFETs as the switching devices, and MUR820G (200 V, 8A) as fast recovery diodes. A battery bank of 4 SDCSs of 60 V dc (nominal) was used to individually supply each inverter level. This prototype was configured to work as 5-, 7-, or 9-level cascaded H-bridge inverter according to the number of activated levels. The switching angles which were obtained from the PSO algorithm were converted into time intervals and stored in an EPROM in the form of look-up tables for all possible harmonic profiles and SDCSs values over their defined range of modulation indices. A real-time controller based on the available MCB-1A

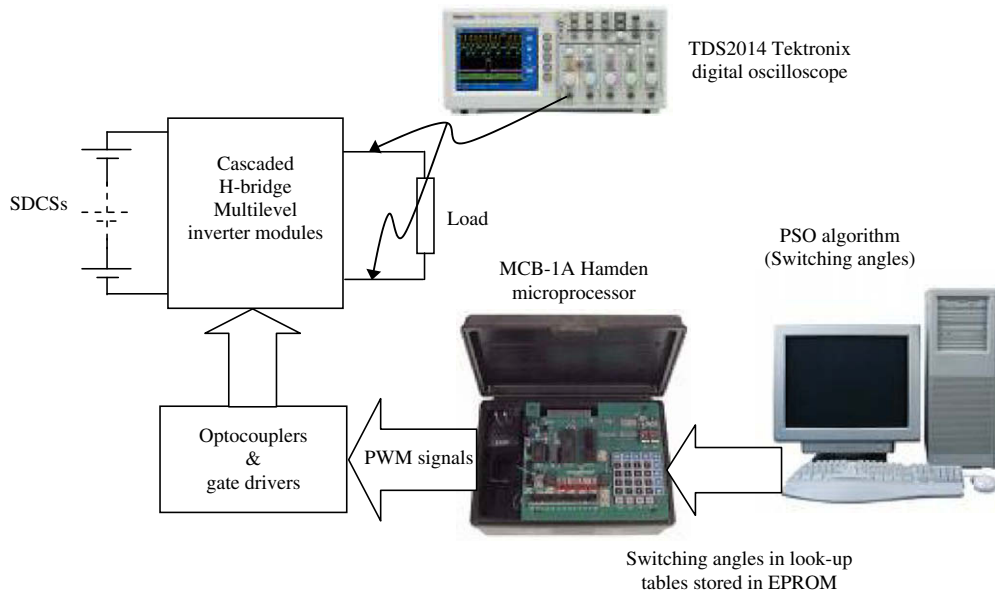


Fig. 14. Schematic diagram of the experimental setup.

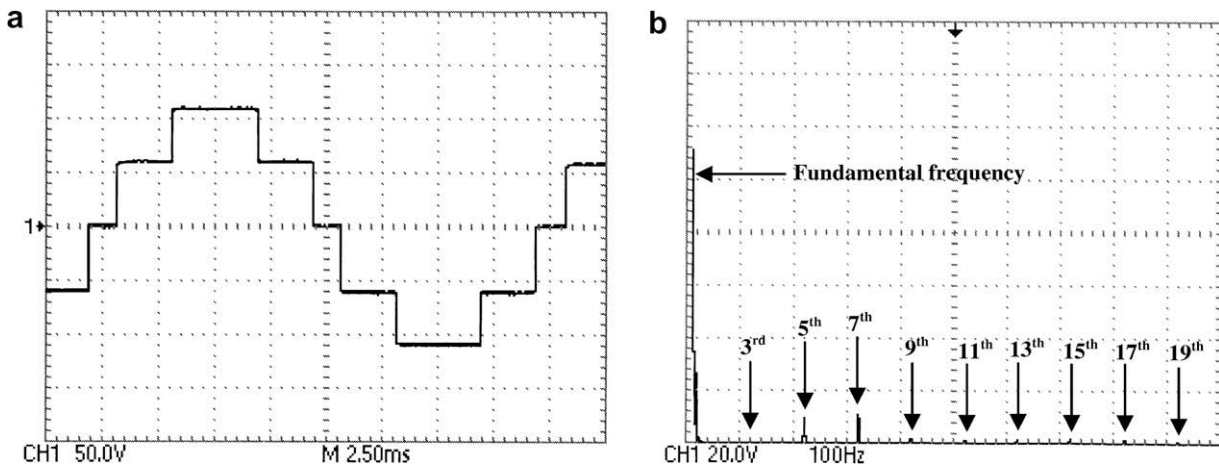


Fig. 15. (a) Output voltage, (b) corresponding FFT of 5-level inverter for case 1 ( $V_{dc} = 60$  V, and  $m = 1.5$ ).

Hampden microprocessor kit was used to implement the harmonic elimination PWM method. The required output voltage of any application such as SVG, UPS, or variable speed drive is mapped to a modulation index, then the appropriate set of switching angles was obtained by cycling through the tables. The switching angles were converted into PWM switching patterns using down-counters and some logic operations, and then were interfaced to the inverter power switches through optocoupler isolators 6N137, and drivers ICL7667. A Tektronix digital oscilloscope TDS2014 with 100 MHz bandwidth and 1 G samples/s was used to display and capture the output waveforms, and with its FFT built-in feature, the spectrum of the output voltage is obtained for different study cases. The schematic diagram of the experimental setup including the microprocessor, interfacing circuits, inverter circuit, and the displaying instrument is shown in Fig. 14. In order to verify the presented simulation results, the hardware implementation was developed for the same study cases of 5-, 7-, and 9-level inverters with the same values of SDCSs as indicated in cases 1, 2, and 3. The experimental inverter output voltage and its corresponding FFT of case 1 regarding 5-level inverter are shown in Fig. 15. The selected value of  $V_{dc} = 60$  V sets  $V_1 = 60$  V and  $V_2 = 54$  V corresponding to 1 p.u. and 0.9 p.u., respectively. Clearly, the spectrum shown in Fig. 15 confirms that  $V_f$  is about 110 V ( $\approx 1.9$  p.u.) and the targeted 3rd harmonic is eliminated. The small deviation between the measured value of  $V_f$  and that from simulations is due to the voltage drop on the semiconductor

devices, as ideal switches were used in simulations. The experimental results of case 2 regarding 7-level inverter are shown in Fig. 16, where  $V_1 = 60$  V,  $V_2 = 54$  V, and  $V_3 = 48$  V. Three switching angles per quarter cycle can be seen, and a fundamental voltage  $V_f$  of about 142 V ( $\approx 2.419$  p.u.) is achieved while the targeted 3rd and 5th harmonics are eliminated. The experimental verification of case 3 regarding 9-level inverter with four switching angles per quarter cycle is shown in Fig. 17. The fundamental voltage  $V_f$  is about 178 V ( $\approx 3.05$  p.u.) and the targeted 3rd, 5th, and 7th harmonics are totally eliminated. It is clearly evident that all experimental results are in close agreement with those from simulations, which reflect the solidness and effectiveness of the presented PSO algorithm. The %THD of the experiments was found a little higher than those of the simulation because the control resolution of the microprocessor is limited 8  $\mu$ s, and the switches are not ideal as in simulations.

**7. Conclusions**

Harmonic elimination of multilevel inverters with non-equal dc voltage sources using a PSO algorithm has been presented. The algorithm is easy to implement as it requires few parameters and no evolution intermediate operators. It overcomes the complicated computations associated with conventional iterative techniques, and the large number of parameters required for GA. It also reduces both the computational burden and running time, and ensures the

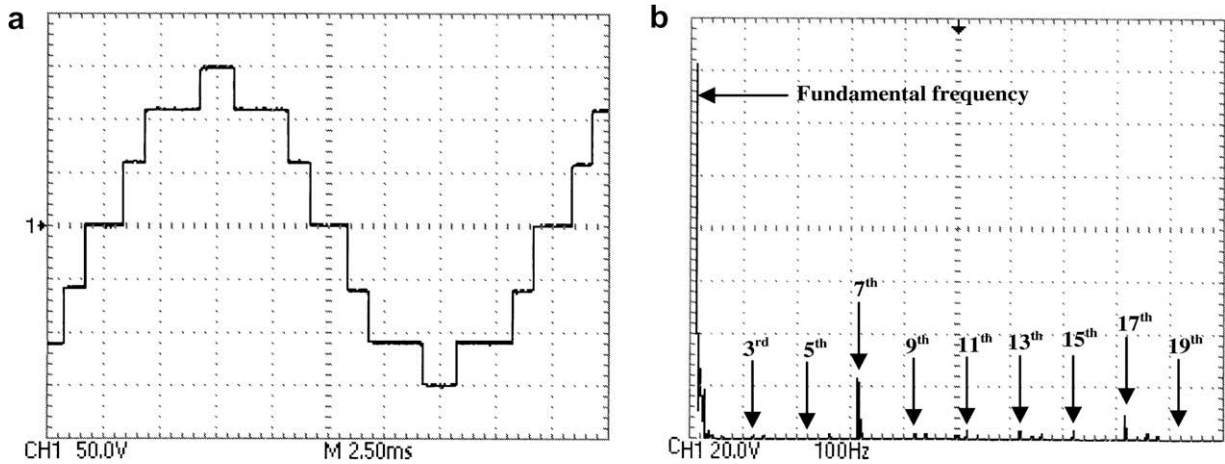


Fig. 16. (a) Output voltage, (b) corresponding FFT of 7-level inverter for case 2 ( $V_{dc} = 60$  V, and  $m = 1.9$ ).

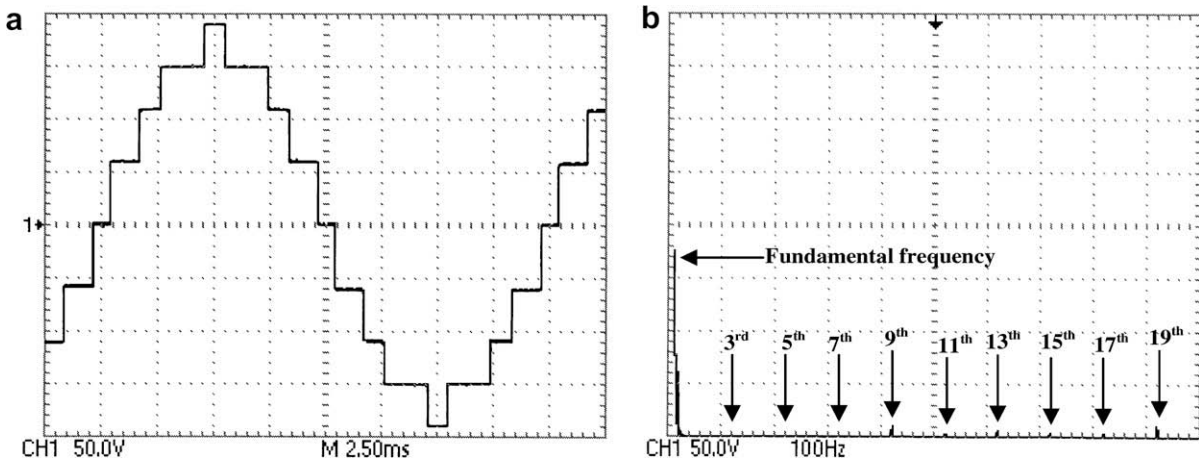


Fig. 17. (a) Output voltage, (b) corresponding FFT of 9-level inverter for case 3 ( $V_{dc} = 60$  V, and  $m = 2.4$ ).



accuracy and quality of the calculated angles. This method was found superior to conventional techniques that may fail to converge if higher levels with non-equal dc sources are sought. In order to prove the feasibility and effectiveness of the proposed algorithm, it is applied to different study cases regarding the number of inverter levels and targeted harmonics to be eliminated. For each case defined by the values of the dc sources and the required harmonic profile, optimal solution can be found over a definite range of modulation index. The PSO technique proved its effectiveness in finding optimal solutions in extremely short time for different values of the dc sources, where THD was taken as a performance index to examine the effectiveness of the solution. Some of these sets of solutions could not be obtained before using traditional techniques. Experimental implementations proved the effectiveness of the proposed method and were in a good agreement with the simulation results. A comparison between the proposed technique and the conventional Newton–Raphson method in terms of computational times and resulted %THD is also reported, where it reveals that the algorithm can be effectively used for selective harmonic elimination of multilevel inverters and results in a dramatic decrease in both the computational times and the output voltage %THD. This method is expected to have widespread applications especially in on-line static var compensation systems, as practical multilevel inverters have non-equal dc sources, and on-line updates of the power system harmonics is highly required.

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