A Semi-definite Relaxation Approach for Partial Discharge Source Location in Transformers

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ABSTRACT
Current issues on localization algorithms based on the time difference of arrival (TDOA) for the partial discharge (PD) source include its sensitivity to time delay error, easy local convergence or divergence, and the large amount of computational load and time. A semi-definite relaxation method for PD source location to solve the time delay positioning equations is proposed in this paper, which using the semi-definite programming problem has the characteristic that can ensure obtained the global optimal solution. The proposed method converts nonlinear time delay equations into an equivalent semi-definite programming (SDP) problem by equivalent transformation and rank-1 relaxation firstly. Then use the interior point algorithm to solve the SDP problem that to obtain a unique global optimum solution, and extract a rank-1 component from the global optimal solution of relaxed SDP. Finally, to serve as a good approximate of the original problem for the PD location. The method was used to localize a measured PD source signal in the laboratory, and the results were compared with those of positioning by the Newton-iterative method. The comparison showed that the method can reduce sensitivity for the time delay error as well as effectively solve TDOA location equations, thereby ensuring that the result is a unique global optimal solution with high positioning efficiency. The localization algorithm problem is solved with the unavoidable and difficult-to-locate time delay error.

Index Terms - **Partial discharge, nonlinear time-difference equations, semi-definite programming, semi-definite relaxation, global optimum solution.**

1 INTRODUCTION

PARTIAL discharge (PD) has been known as the “insulation tumor”. If it can be detected early and effectively governed, it becomes an effective method to ensure the insulation equipment safe operation. If the location of the PD source and its discharge quantities can be obtained quickly and accurately, not only the insulation condition of the electrical equipment can be determined, but also potential fault can be eliminated and long-term safe operation can be ensured.

Ultra high frequency (UHF) detection technology, which has the advantages of good anti-jamming ability and high sensitivity in transformer applications, is a focus of concern in the field of PD source location [1-3]. The implementation of the time difference of arrival (TDOA) method in locating the PD source is to access accurately the PD signal using four or more sensors that are installed and distributed on the transformer tank for simultaneous recording of PD signals, obtaining the differences in the time the signals reach each sensor (also called the time delay), and identifying the time delay equations which need to be solved to determine the spatial location of the PD sources [4]. The UHF technology therefore locates the PD source in a transformer by performing two key steps, namely, accurately accessing time delay and by solving nonlinear time delay

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equations. When the positioning equation becomes extremely sensitive to the calculation of the time delay error, small time errors can result in high location errors and time difference equations can derive no solutions and produce a false location result. In practice, however, time delay normally causes measurement errors or uncertainties that require more stringent requirements for localization algorithms.

The spherical or hyperbolic time delay equations are nonlinear and cannot extract the root formula, which exacerbates the difficulty of positioning transformer requirements accurately and quickly. Thus, previous studies had sought to improve the localization algorithm to promote location accuracy and proposed a number of effective optimal solution methods [5-8]. For example, reference [9] used the Newton iteration method to solve time difference equations and achieved high positioning accuracy when the initial value was close to the real value. In [10], the global search capability of the genetic algorithm for nonlinear time difference equations was used to achieve PD source localization. In [11], the particle swarm optimization (PSO) algorithm was employed to solve positioning equations because it can effectively address the problem of the traditional algorithm in which a single search mechanism causes the stagnation phenomenon and avoids oscillation or divergence at the boundary of the algorithm. However, all obtained algorithms encounter the phenomenon that traps the local optimal solution during calculation, leading to a large location error and causes difficulty in obtaining accurately the PD fault location. In [12], the study proposed dividing the entire transformer space into several grids to locate the global optimal solution by the grid search method and to effectively avoid the local convergence or divergence problem. The disadvantages are more complex, resulting in a large amount of computational load and time. A number of time delay measurements must be computed during factual operation to improve the credibility and accuracy of the positioning results, subsequently increasing the requirements for the computation of the localization algorithm. The present TDOA positioning algorithm has three major difficulties, namely, its sensitivity to time delay error, easy local convergence or divergence, and a large amount of computational load and time [13]. To some extent, these shortcomings restrict the application of UHF technology in transformer PD source positioning.

In recent years, with the development and maturity of the semi-definite programming (SDP) theory [14-16], an increasing amount of attention has been given to the application of the SDP in the field of signal processing. Compared with other optimization algorithms, the most distinguishing characteristic of the SDP is that has the global optimal solution and unique. To solve issues in the present localization algorithm, a semi-definite relaxation (SDR) method that can solve positioning equations is proposed in this paper. The SDR method converts the nonlinear time delay equation problem of the difficult-to-solve PD source position to an equivalent non-convex SDP problem by equivalent transformation and rank-1 relaxation. The SDP problem after relaxation becomes easy to solve and obtains a unique global optimal solution. The software of *SeDuMi* 1.3, which is a MATLAB toolbox for solving the SDP problem, and giving the calculation method of about how to extract PD source location estimation from the optimal solution. The method is validated by the measured data of the experimental research and then compared with those of the grid search and the Newton iteration methods to analyze the efficiency of the various methods.

2 PROPOSED MATHEMATICAL PROBLEM FOR PD SOURCE LOCALIZATION

When PD failure occurs in a transformer, the PD source (assumed to be located at $P=[x,y,z]^{\mathrm{T}}$) will radiate UHF signals that will spread in all directions as a form of spherical wave from the origin location. The PD signals will then be received simultaneously by four UHF sensors (assumed to be installed at $S_i=[x_i,y_i,z_i]^{\mathrm{T}}$, $i=0,1,2,3$) placed at arbitrary locations, as shown in Figure 1. Given S_0 as a reference antenna and its coordinate is $S_0=[0,0,0]^{\mathrm{T}}$, the distance of the sensors S_i or source P to the origin of reference coordinate system can be shown as

$$R_i=\|S_i\|=\sqrt{x_i^{\,2}+y_i^{\,2}+z_i^{\,2}}\,,\ i=1,2,3 \tag{1}$$

$$R_P=\|P\|=\sqrt{x^2+y^2+z^2} \tag{2}$$

where $\|\bullet\|$ is the vector 2-norm.

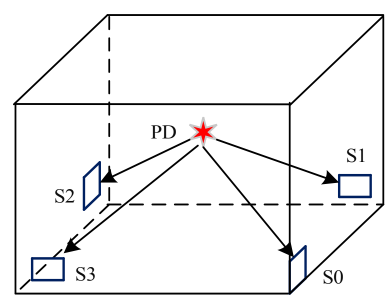


Figure 1. Principle figure of four sensors locating the PD source.

Suppose the time when UHF signals are generated by the PD source at constant speed and are travelling in straight lines to S_0 is represented as T, τ_{i0} is the time taken by the PD signal to arrive between the i^{th} sensor and S_0, and when i=0, $\tau_{i0}=0$. According to the working principle of the TDOA technique [17], the time delay equations can be established as follows:

$$\begin{cases}(x-0)^2+(y-0)^2+(z-0)^2=v^2T^2\\(x-x_1)^2+(y-y_1)^2+(z-z_1)^2=v^2(T+\tau_{10})^2\\(x-x_2)^2+(y-y_2)^2+(z-z_2)^2=v^2(T+\tau_{20})^2\\(x-x_3)^2+(y-y_3)^2+(z-z_3)^2=v^2(T+\tau_{30})^2\end{cases} \tag{3}$$

where the coordinates $S_i=[x_i,y_i,z_i]^{\mathrm{T}}$ of the four sensors and the wave velocity v are known.

After obtaining the time delay, which is calculated by the sensor-acquired PD signals and by solving the time delay equations (3), the location of the PD source can be obtained. However, these equations are nonlinear and are difficult to solve directly in the traditional method. The equations can be transformed into an optimization problem with constraints. Thus, Equation (3) can be simplified as

$$d_i = D_i - D_0 = v \bullet \tau_{i0} \tag{4}$$

where D_i is the distance between the PD source and the i^{th} sensor, that is,

$$D_i = \| S_i - P \| = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \tag{5}$$

where $D_0 = R_P$. Therefore, using Equations (1), (2), (4), and (5), we can prove that

$$S_i^{\text{T}} P + d_i R_P = \frac{1}{2}(R_i^2 - d_i^2) \tag{6}$$

Equation (6) can be written in matrix form as follows:

$$AX = K \tag{7}$$

where $A = \begin{bmatrix} x_1 & y_1 & z_1 & d_1 \\ x_2 & y_2 & z_2 & d_2 \\ x_3 & y_3 & z_3 & d_3 \end{bmatrix}$, $X = \begin{bmatrix} P \\ R_P \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ R_P \end{bmatrix}$ and $K = \frac{1}{2}\begin{bmatrix} R_1^2 - d_1^2 \\ R_2^2 - d_2^2 \\ R_3^2 - d_3^2 \end{bmatrix}$.

However, in practical applications, time delay measurement errors and uncertainties are inevitable, resulting in the actual measurement value $\hat{d}_i = v \bullet \hat{\tau}_{i0} = d_i + \varepsilon_i$, where $\hat{\tau}_{i0}$ is the time difference of actual measurement and ε_i is the distance error caused by the time delay error. Equation (7) can be expressed as

$$e = AX - K \tag{8}$$

where e is the equation error caused by the distance error. Under the criterion of minimizing the square of equation error, and considering about the constraint relationship $R_P = \|P\|$ for X, the PD source location estimation problem can be described as the following optimization problem:

$$\begin{cases} \min\limits_{X} . \ \|AX - K\|^2 \\ \text{s.t.} \ \ X^{\text{T}} \Lambda X = 0 \end{cases} \tag{9}$$

where $\Lambda = \text{diag}(1,1,1,\text{-}1)$. Thus, the equation can be solved using mathematical optimization methodology to obtain the optimal solution for the optimization problem (9), which can determine the spatial location of the PD source.

3 SEMI-DEFINITE RELAXATION LOCATION METHOD FOR THE PD SOURCE

3.1 INTRODUCTION OF SEMI-DEFINITE PROGRAMMING

SDP [18, 19] is a mathematical programming method which originated in the 1990s and is defined as an extension of linear programming, with vector variables replaced by matrix variables and non-negativity element-wise replaced by positive semi-definiteness. In linear SDP, one maximizes or minimizes a linear function relative to the constraint that an affine combination of symmetric matrices is positively semi-definite. This constraint is non-linear and non-smooth but convex. Thus, The SDP is a nonlinear convex optimization problems. The standard form of the SDP can be expressed as

$$\begin{cases} \min . \ \text{tr}(CX) \\ \text{s.t.} \ \ \text{tr}(A_i X) = b_i \ , i = 1,2,...,m \\ \qquad X \succeq 0 \end{cases} \tag{10}$$

where X is the optimization variable of the non-negativity constraint matrix with n order, C, A_i are the data parameters of real symmetric matrices with n order, tr(•) represents the trace of the matrix, $b_i (i = 1,2,...,m)$ is the real number, and the symbol "$\succeq$" expresses semi-definiteness that indicate X as a positive semi-definite matrix.

Optimization problem (9) is a non-convex quadratic optimization problem with quadratic equality constraint. Hence, this paper can employ (9) into an equivalent SDP problem (10) to obtain a solution.

3.2 METHOD OF SEMI-DEFINITE PROGRAMMING RELAXATION

Semi-definite relaxation [20] is a precise approximation method in solving nonlinear, non-convex optimization problems, which can obtain the optimal solution for the original optimization problem. An equivalent transformation is considered for the objective function of problem (9), and the original optimization problem is equal to

$$\min_{X} . \begin{bmatrix} X^{\text{T}} & 1 \end{bmatrix} \begin{bmatrix} A^{\text{T}}A & -A^{\text{T}}K \\ -K^{\text{T}}A & K^{\text{T}}K \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix} \tag{11}$$

According to the nature of the traces of matrix $x^{\text{T}}Bx = \text{tr}(x^{\text{T}}Bx) = \text{tr}(xx^{\text{T}}B)$, (11) can be represented as

$$\begin{cases} \min\limits_{\theta} . \ \text{tr}\left(\begin{bmatrix} X \\ 1 \end{bmatrix} \begin{bmatrix} X^{\text{T}} & 1 \end{bmatrix} \begin{bmatrix} A^{\text{T}}A & -A^{\text{T}}K \\ -K^{\text{T}}A & K^{\text{T}}K \end{bmatrix} \right) \\ \text{s.t.} \ \ \text{tr}\left(\begin{bmatrix} X \\ 1 \end{bmatrix} \begin{bmatrix} X^{\text{T}} & 1 \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix} \right) = 0 \end{cases} \tag{12}$$

In the assumption that $G = XX^{\text{T}}$, the optimization problem (12) is equivalent to

$$\min_{X,G} . \ \text{tr}\left(\begin{bmatrix} G & X \\ X^{\text{T}} & 1 \end{bmatrix} \begin{bmatrix} A^{\text{T}}A & -A^{\text{T}}K \\ -K^{\text{T}}A & K^{\text{T}}K \end{bmatrix} \right) \tag{13a}$$

$$\text{s.t.} \ \ \text{tr}\left(\begin{bmatrix} G & X \\ X^{\text{T}} & 1 \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix} \right) = 0 \tag{13b}$$

$$\begin{bmatrix} G & X \\ X^{\text{T}} & 1 \end{bmatrix} \succeq 0, \ \text{rank}\left(\begin{bmatrix} G & X \\ X^{\text{T}} & 1 \end{bmatrix} \right) = 1 \tag{13c}$$

Here, the equivalence relation is used in equation (13c) as follows:

$$G = XX^{\mathrm{T}} \Leftrightarrow \begin{bmatrix} G & X \\ X^{\mathrm{T}} & 1 \end{bmatrix} \succeq 0,\ \mathrm{rank}\left(\begin{bmatrix} G & X \\ X^{\mathrm{T}} & 1 \end{bmatrix}\right) = 1 \qquad (13d)$$

where $\mathrm{rank}(\bullet) = 1$ denotes that the rank of the matrix is 1. Equation (13c) is a non-convex constraint, thus problem (13) remains as a non-convex optimization problem. In transforming non-convex optimization problem (13) into a SDP problem that can consider rank-1 relaxation for (13c), the optimization problem after relaxation is

$$\begin{cases} \min\limits_{X,G}.\ \mathrm{tr}\left(\begin{bmatrix} G & X \\ X^{\mathrm{T}} & 1 \end{bmatrix}\begin{bmatrix} A^{\mathrm{T}}A & -A^{\mathrm{T}}K \\ -K^{\mathrm{T}}A & K^{\mathrm{T}}K \end{bmatrix}\right) \\ \text{s.t.}\quad \mathrm{tr}\left(\begin{bmatrix} G & X \\ X^{\mathrm{T}} & 1 \end{bmatrix}\begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix}\right) = 0 \\ \qquad \begin{bmatrix} G & X \\ X^{\mathrm{T}} & 1 \end{bmatrix} \succeq 0 \end{cases} \qquad (14)$$

In the convex programming theory, Equation (14) represents SDP. The global optimization solution G^* can be obtained by solving Equation (14), which is the optimal solution for problem (9), and the first three values specify the PD source position in $X^* = \left[x^*, y^*, z^*, R_P^*\right]^{\mathrm{T}}$.

3.3 CALCULATION OF THE COORDINATES OF THE PD SOURCE

After the previously abandoned rank-1 constraint for problem (9) has been transformed equivalently based on the SDR method, the rank-1 constraint of optimization problem (9) for PD source location can be restored after obtaining optimal solution G^* from solving equation (14). Rank-1 component XX^{T} must be extracted from the global optimum solution and serve as a good approximate of the original problem (9). The following two situations must be discussed:

(1) When $\mathrm{rank}(G^*) = 1$, the rank-1 constraint of problem (9) is satisfied. In this case, the optimal solution can be obtained by $G^* = X^* X^{*\mathrm{T}}$, and the location P^* of coordinates within X^* is the PD source position that must be estimated.

(2) When $\mathrm{rank}(G^*) > 1$, the constraints are not matched with problem (9). Thus, X^* necessarily performs rank-1 approximation for G^*. The approximate solution method is as follows.

First, based on the eigenvalue decomposition for G^*, we can obtain

$$G^* = \sum_{i=1}^{4} \lambda_i \upsilon_i \upsilon_i^{\mathrm{T}} \qquad (15)$$

where λ_i is the eigenvalue of G^* that satisfies $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ and υ_i is the corresponding eigenvectors of λ_i. $\sqrt{\lambda_i}\upsilon_i$ is projected onto the feasible region of equation (9) to satisfy the constraint of $f(\upsilon_1)^{\mathrm{T}} \Lambda f(\upsilon_1) = 0$ and to obtain the approximate optimal solution $X^* = f(\upsilon_1)$ of problem (9). Consequently, the optimal location of PD source can be determined, where $f(\bullet)$ expresses the projection operation.

Specific steps are as follows: $h = \lambda_1 \times \upsilon_1 \odot \upsilon_1$ and $q = p \odot p$ are set, in which $\odot$ is the Schur product of the vector. According to [21], the least squares of q can be obtained as

$$\hat{q} = \frac{1}{4}\begin{bmatrix} 3 & -1 & -1 & 1 \\ -1 & 3 & -1 & 1 \\ -1 & -1 & 3 & 1 \end{bmatrix} \times h \qquad (16)$$

The approximate optimal solution of problem (9) can be shown as

$$\begin{aligned} X^* &= f(\upsilon_1) \\ &= \mathrm{diag}(\mathrm{sgn}(\upsilon_1)) \times \left[\sqrt{\hat{q}(1)}, \sqrt{\hat{q}(2)}, \sqrt{\hat{q}(3)}, \upsilon_1(4)\right]^{\mathrm{T}} \end{aligned} \qquad (17)$$

where $\mathrm{sgn}(\upsilon_1)$ represents the symbols of each element of the principal eigenvectors.

4 ANALYSIS OF THE EXPERIMENTAL RESULTS

4.1 EXPERIMENTAL SETUP

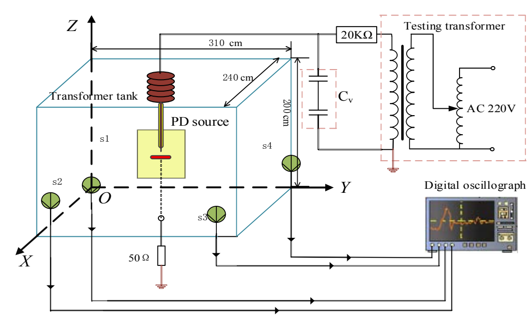


Figure 2. Experimental system for PD location.

Transformer simulation experimental setup was prepared to test the PD source locus effect on accuracy of the SDR location method on PD source. The investigation was carried out in laboratory to measure UHF signals and to locate the PD source using the test setup, as shown in Figure 2. The PD source was generated by a needle-plate structure discharge model and PD occurs in a 40 cm diameter and 50 cm high plexiglass casing, which is filled with insulation oil (Pale yellow part). And the casing can be placed anywhere inside the tank, thus facilitating the convenient study of the positioning of the PD source at different places. Because of the tank is huge, it is difficult to fill so much oil into it. In addition, the experiment often need to be change the locations of PD source and sensors, so the rest of tank with air as the medium, which not only experimental set-up convenience, but

Table 1. Date of measured time delay and calculated PD location.

NO. i	Original location (cm)	Theoretical value /ns	Measured value /ns	Time error /ns	SDR method		NR method	
					Coordinates /cm	Error /cm	Coordinates /cm	Error /cm
1	(41, 56, 102)	3.00, 6.82, 4.89	2.85, 6.71, 4.83	-0.15, -0.11, -0.06	(42, 54, 109)	7.35	(48, 53, 108)	9.70
2	(136, 83, 89)	-1.31, 2.36, 3.03	-1.52, 2.24, 3.06	-0.21, -0.12, 0.03	(140, 83, 82)	8.06	(102, 89, 106)	38.48
3	(163, 226, 89)	-1.56, -5.32, -3.22	-1.25, -4.83, -3.27	0.31, 0.49, -0.05	(152, 225, 88)	11.09	(181, 197, 62)	43.52
4	(52, 126, 120)	1.94, 3.24, 1.14	1.66, 3.09, 1.25	-0.28, -0.15, 0.11	(59, 123, 126)	9.70	(35, 148, 156)	45.49
5	(66, 231, 120)	1.06, -1.71, -4.17	0.65, -1.91, -4.46	0.41, -0.20, -0.29	(74, 233, 127)	10.82	(71, 207, 139)	31.02

also simulated the positioning research of PD signal in two medium (oil and air). And in order to simulate diffraction phenomenon of UHF signals in transformer, we put some insulators on the sensor receiving side as the obstacles. A WavePro 7100XL oscilloscope with four channels (analog band: 1 GHz; sampling rate: 20 GHz; memory depth: 48 MB) was used to simultaneously capture PD signal data detected from four UHF micro-strip patch sensors installed on a large transformer box body (240 cm × 310 cm × 200 cm), which determined the time delay [22]. The reference coordinate system is shown in Figure 2, and the space coordinates of the sensors are S_0(0,0,0), S_1(230,0,20), S_2(240,300,10), and S_3(0,310,20) cm.

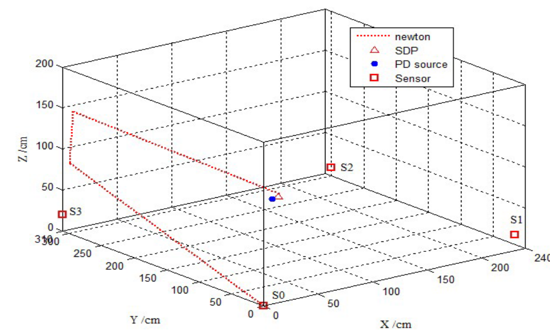
This paper uses the SDR and the Newton-Raphson (NR, also called Newton iterative) methods to calculate five different places of the PD source and to analyze the localization effect and the advantages and disadvantages of the measured time delay error that can manifest in the experiment. The time difference was calculated using the time difference on an energy accumulation curve of multiple signals. The method is that uses the mathematical transformation of "relevant-shift-superposition" to obtain a statistical accumulated energy curve from a large amount of PD samples in laboratory. The time delay of multiple sensors by relevant search principle was obtained (The principle and formula can see in reference [23]). The *SeDuMi* toolbox [24] was used to solve the SDP problem, and the conditions of the iteration ending of the NR method was established as follows: number of iterations is 100 and precision is $\varepsilon = 10^{-1}$. The time delays of the theoretical value, measured value, and the calculation results of the two positioning methods for five PD sources are shown in Table 1.

4.2 ANALYSIS OF LOCALIZATION RESULTS

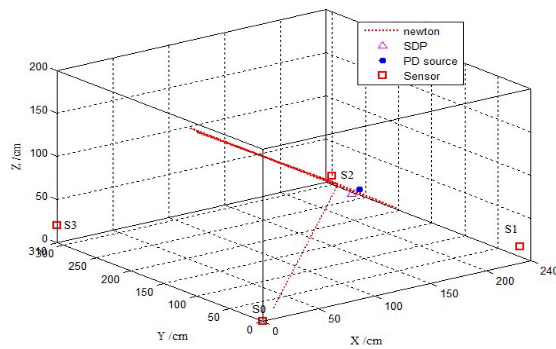
The calculation processes of Nos. 1 and 2 discharge sources were considered in the analysis to compare the convergent characteristic of the two algorithms. The convergence curve is shown in Figure 3.

4.2.1 CONVERGENCE

Figure 3a shows the results in every step of the iterative process in the location calculation for the No.1 source and that when the NR method was used to calculate, the starting point of the iteration was *O*(0,0,0), which moved along the +*Y* and +*Z* directions and then to the -*Y* direction until finally converging to point (48,53,108). For the SDR method, the result was (42, 54, 109). Both methods all obtained the



(a) No. 1 Discharge source



(b) No. 2 Discharge source

Figure 3. Convergence curve of SDR and NR methods.

approximate solutions because the time delay measurement error was smaller for the No.1 source. Hence, both algorithms achieved the convergence conditions and obtained the approximate solution.

Figure 3b shows the results in every step of the iterative process in the location calculation for the No.2 source. When the NR method was used, the iterative process began near point *O* and moved to +*X*, +*Y*, and +*Z* directions. However, the algorithm did not converge and the reciprocation moved parallel to the *Y* direction until the terminating condition of iterative 100 times the value of the location result was satisfied and the local optimal solution was determined. The iteration result was (102, 89, 106). When the SDR method was used for the location, a unique global optimal solution was obtained and the result was (140, 83, 82). The differences in the results of the various methods were due to the time delay error that caused the positioning equations to derive no solution. The NR method required a suitable initial value of iteration to be transformed into the local optimal solution, resulting in a poor location effect. The SDR method, with its advantages, successfully obtained an approximate solution.

4.2.2 POSITIONING ACCURACY

Table 1 shows that because NR and SDR methods were convergent, the global optimal solution for the No. 1 PD source can be obtained. The positioning accuracy of both methods was consistent and the results identified the location of the PD source. For the No. 2 source, because the Newton iterative calculation did not converge, the result error obtained after 100 iterations was considerably large, reaching 31.65 cm. When using the SDR method, the global optimal solution can be obtained with slight deviation at 8.06 cm. The SDR method demonstrated higher accuracy than the NR method and its positioning accuracy met the requirements of transformer localization.

A comparison between the time delay errors of the No.1 and the No. 2 PD sources shows that although both errors were negligible, a large difference in the localization results was observed in the NR method. This difference illustrates that the present localization algorithm is extremely sensitive to time delay error, and that small errors may lead to a larger errors or a divergence in the localization result exists, consequently leading to the failure to locate. By contrast, the time delay error was less affected by the SDR method than that by the NR method. Thus, the localization effect of the SDR method is better than that of the NR method.

The same results were obtained for the Nos. 3, 4, and 5 PD sources. The equations obtained no solutions due to the time delay error, and the use of the NR method resulted in only the local optimal solution, causing a large error in the positioning result and inaccurate identification of the location of the PD source. In using the SDR method, a unique global optimum solution of the equations can be obtained and the location of the PD source can be determined. The statistics in Table 1 shows that the average location error was 33.64 cm when the NR method was used and 9.04 cm when the SDR method was used. Thus, the positioning result obtained by the SDR method had higher precision, with a value closer to the PD source location, than that by the NR method. This result indicates that the nonlinear localization problem had been transformed into a SDP problem that can ensure that the positioning result is a unique global optimal solution, so as to avoid the possible issues of local convergence or divergence obtained by the other methods. The results further indicate that the SDR method can reduce sensitivity to the time delay error and improve the reliability of localization.

4.3 ANALYSIS OF LOCATION EFFICIENCY

The accuracy and the rapid acquisition of positioning results by various methods were compared by calculating, as an example, the location of the No. 2 source, which is on the same computer platform. The Semi-definite Relaxation (SDR), Grid Search (GS), and Newton-Raphson (NR) methods were employed to compare the time needed for computation. For the GS method, the size of the mesh was divided into the following cases: GS1 (10 cm × 10 cm × 10 cm) and GS2 (5 cm × 5 cm × 5 cm). For the NR method, four cases were selected, which include NR1 = (the number of iterations is 100, precision is $\varepsilon = 10^{-1}$), NR2 = (the number of iterations is 1000, precision is $\varepsilon = 10^{-1}$), NR3 = (the number of iterations is 1000, precision is $\varepsilon = 10^{-4}$), and NR4 = (the number of iterations is 10000, precision is $\varepsilon = 10^{-4}$). The comparison results, which are the average of the results of 10 computations for each case that correspond to the positioning results and to the calculation time, are shown in Table 2.

Table 2 shows that the GS method can determine the PD source location accurately. Positioning accuracy increased as grids decreased, but the time required and the costs rose sharply. The time spent to obtain the same level of positioning accuracy is more than 300 times than that of the SDR method. For the Newton iteration method, when the number of iterations is 100 and precision is $\varepsilon = 10^{-1}$, the algorithm does not converge, and the value of the 100th time as the positioning result is selected, causing the positioning error to increase. Along with the increase in iteration times and the decrease in convergence accuracy, the values are similar to those of the original location of the PD source. Under a large time delay error, the time required by the Newton iteration method is approximately 400 times compared with that of the SDR method to achieve the same level of positioning accuracy as the SDR method. A large amount and period of calculation is necessary because the iteration requires the values of function and the derivative to be obtained. Obviously, the SDR method can locate faster than the other two methods.

Table 2. Comparison between different methods.

Methods	Location Result (cm)	Error (cm)	Execution time (s)
SDR	(140, 83, 82)	8.06	7.64
GS1	(147, 88, 78)	16.34	659.43
GS2	(139, 84, 83)	6.78	2520.93
NR1	(102, 89, 106)	38.48	13.36
NR2	(114, 91, 93)	23.75	117.73
NR3	(132, 86, 98)	10.30	1272.19
NR4	(143, 85, 86)	7.87	2944.26

The results indicate that the SDR method has a particular advantage in locating the transformer PD, which is the transformation of the local minimum problem to the SDP problem, thereby avoiding reduction of the time necessary for the location results to achieve the global optimal solution. The process is likewise beneficial in forming an accurate judgment of the location of the PD source. The experiments show that after converting nonlinear localization equations to an equivalent SDP problem, the rank of the optimal solution is always equal to 1, making rank-1 approximation unnecessary and allowing the optimal solution of the original problem to be obtained directly. The SDR method likewise avoids the error in rank-1 approximation that can influence the positioning result.

4.4 DISCUSSION

There is a complicated structure in real transformer with the iron cores, windings and so on. And yet, the experimental simulation model cannot reflect the actual complex structure of the transformer, which is also an indispensable part of the experimental platform. Therefore, future works should simulate the real structure of the transformer. Studies should also explore the mechanism of electromagnetic wave propagation, leakage inside the transformer, effect of non-linear propagation signal in multi-layered media on positioning accuracy, and the influence of sensor number (more than four) on the location. Finally, future research should verify the feasibility and veracity of the proposed method.

5 CONCLUSION

(1) Localization of the UHF partial discharge sources using UHF in transformers has several inevitable time difference measurement errors and uncertainties. However, the present localization algorithm is considerably sensitive to time delay error, resulting in premature convergence and being confined in the local optimum that leads to large positioning error. Likewise, the location of the PD source cannot be determined accurately and the location efficiency is low.

(2) SDP is a convex optimization problem that can be solved by a unique global optimal solution. This paper converts TDOA equations to an SDP problem in identifying the PD source location by equivalent transformation and rank-1 relaxation. The conversion proved effective in solving the problems of present localization algorithms (such as Newton iterative method, Genetic Algorithm, and so on) in terms of the difficulty to locate.

(3) The SDR method can solve the TDOA location equations quickly, succinctly, and with high positioning efficiency. The calculation results can accurately estimate the location of the PD source and have good reliability. The location result of the experimental data shows that compared with previous algorithms, the method based on SDR can effectively, quickly, and accurately locate the PD source.

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